

Exploring the proton spin structure

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Abstract Understanding the spin structure of the proton is one of the main challenges in hadronic physics. While the concepts of spin and orbital angular momentum are pretty clear in the context of non-relativistic quantum mechanics, the generalization of these concepts to quantum field theory encounters serious difficulties. It is however possible to define meaningful decompositions of the proton spin that are (in principle) measurable. We propose a summary of the present situation including recent developments and prospects of future developments.

1 Introduction

Understanding how the proton spin arises from the spin and orbital motion of its constituents is one of the most challenging key questions in hadronic physics. Hadrons are very peculiar physical systems as their constituents are highly relativistic and confined. One has therefore to use cunning in order to unravel their internal structure.

While it is clear that a proton at rest has total angular momentum $J = 1/2$, the decomposition of the latter in terms of spin and orbital contributions associated with quarks and gluons is not unique, creating some confusion and raising serious controversies among physicists. Most of the discussions focused on determining which one of the proposed decompositions has to be considered as the “physical” or fundamental one. Now that the dust has settled, it turns out that the angular momentum decomposition is intrinsically ambiguous because of Lorentz and gauge symmetry. However, this does not imply that the question of the angular momentum decompo-

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sition does not make sense at all, but rather emphasizes the fact that the description of a physical phenomenon does not need to be unique. What is considered as the “physical” or fundamental description usually turns out to be the simplest or most convenient description at hand.

While measurable quantities are necessarily gauge invariant, it has recently been recognized that they need not be local or *manifestly* Lorentz covariant. Departing from locality or manifest Lorentz covariance leads to ambiguities as there exists in principle infinitely many ways to do so. What saves the day is that it is the way the physical system is probed, *i.e.* the experimental configuration, which determines the natural or sensible departure from locality or manifest Lorentz covariance. For example, the internal structure of the proton is essentially probed in high-energy experiments which provide us with a natural preferred direction breaking manifest Lorentz covariance. This preferred direction can then be used to define the natural angular momentum decomposition.

One of the crucial questions now is to identify the experimental observables from which the orbital angular momentum (OAM) can be extracted. Many different relations and sum rules have been proposed in the last two decades, creating some sort of confusion. One of the remaining tasks consists in clarifying the validity and scope of these relations and sum rules.

In this contribution, we summarize the present situation and mention some recent developments. In section 2, we briefly discuss the two families of proton spin decompositions. In section 3, we collect various spin sum rules and relations. In section 4, we introduce the notion of quark spin-orbit correlation and show how it is related to measurable parton distributions. Finally, we collect our conclusions in section 5. For the interested reader, more detailed discussions can be found in the recent reviews [24, 50].

2 Kinetic and canonical spin decompositions

There are essentially two types of decompositions of the proton spin operator: kinetic (also known as mechanical) and canonical. These two types differ by how the OAM operator is split into the quark (q) and gluon (G) contributions

$$\begin{aligned}\mathbf{J} &= \mathbf{S}^q + \mathbf{L}_{\text{kin}}^q + \mathbf{L}_{\text{kin}}^G + \mathbf{S}^G, \\ &= \mathbf{S}^q + \mathbf{L}_{\text{can}}^q + \mathbf{L}_{\text{can}}^G + \mathbf{S}^G,\end{aligned}\tag{1}$$

where

$$\begin{aligned}\mathbf{S}^q &= \int d^3r \psi^\dagger \frac{1}{2} \boldsymbol{\Sigma} \psi, & \mathbf{S}^G &= \int d^3r \mathbf{E}^a \times \mathbf{A}_{\text{phys}}^a, \\ \mathbf{L}_{\text{kin}}^q &= \int d^3r \psi^\dagger (\mathbf{r} \times i\mathbf{D}) \psi, & \mathbf{L}_{\text{kin}}^G &= \mathbf{L}_{\text{can}}^G - \int d^3r (\mathbf{D}^{ab} \cdot \mathbf{E}^b) \mathbf{x} \times \mathbf{A}_{\text{phys}}^a, \\ \mathbf{L}_{\text{can}}^q &= \int d^3r \psi^\dagger (\mathbf{r} \times i\mathbf{D}_{\text{pure}}) \psi, & \mathbf{L}_{\text{can}}^G &= - \int d^3r E^{aj} (\mathbf{r} \times \mathbf{D}_{\text{pure}}^{ab}) A_{\text{phys}}^{bj}.\end{aligned}\tag{2}$$

$\mathbf{J}_{\text{kin}}^q$		$\mathbf{J}_{\text{kin}}^G$	
\mathbf{S}^q	$\mathbf{L}_{\text{kin}}^q$	$\mathbf{J}_{\text{kin}}^G$	
\mathbf{S}^q	$\mathbf{L}_{\text{kin}}^q$	$\mathbf{L}_{\text{kin}}^G$	\mathbf{S}^G
\mathbf{S}^q	$\mathbf{L}_{\text{can}}^q$	$\mathbf{L}_{\text{can}}^G$	\mathbf{S}^G

Fig. 1 The proton spin decompositions. The first three are the Belinfante, Ji and Wakamatsu versions of the kinetic decomposition. The last one is the Chen *et al.* canonical decomposition which is the manifestly gauge-invariant version of the Jaffe-Manohar decomposition. The contributions in gray are nonlocal or not manifestly Lorentz covariant.

The gauge field has been decomposed into two parts $\mathbf{A} = \mathbf{A}_{\text{pure}} + \mathbf{A}_{\text{phys}}$ where \mathbf{A}_{pure} is a pure-gauge potential. The pure-gauge covariant derivatives are then given by $\mathbf{D}_{\text{pure}} = -\nabla - ig\mathbf{A}_{\text{pure}}$ and $\mathbf{D}_{\text{pure}}^{ab} = -\nabla\delta^{ab} - gf^{abc}\mathbf{A}_{\text{pure}}^c$. A nice physical interpretation of the difference $\mathbf{L}_{\text{pot}} = \mathbf{L}_{\text{kin}}^q - \mathbf{L}_{\text{can}}^q = \mathbf{L}_{\text{can}}^G - \mathbf{L}_{\text{kin}}^G$, known as the potential OAM, has been proposed in Ref. [6].

The complete gauge-invariant kinetic and canonical decompositions (1) are known in the literature as the Wakamatsu [48, 49] and Chen *et al.* [7, 8] decompositions, respectively. The Chen *et al.* decomposition can be seen as a gauge-invariant version (or extension) of the Jaffe-Manohar decomposition [14]. These complete gauge-invariant decompositions seem to contradict textbook claims about the impossibility of separating in a gauge-invariant way the gluon angular momentum into spin and OAM contributions. This impossibility is circumvented by introducing the non-local fields \mathbf{A}_{pure} and \mathbf{A}_{phys} [29, 30], where the pure-gauge field \mathbf{A}_{pure} plays the role of a background field [31, 32]. Background dependence then implies that the decomposition $\mathbf{A} = \mathbf{A}_{\text{pure}} + \mathbf{A}_{\text{phys}}$ comes with a new freedom

$$\mathbf{A}_{\text{pure}} \mapsto \mathbf{A}_{\text{pure}} + \mathbf{B}, \quad \mathbf{A}_{\text{phys}} \mapsto \mathbf{A}_{\text{phys}} - \mathbf{B}, \quad (3)$$

referred to as the Stueckelberg symmetry [29, 47], making the decompositions ambiguous as *a priori* any pure-gauge field \mathbf{A}_{pure} can be used. This issue is however solved by noting that the actual experimental conditions determine the form of the background field to be used [29, 51].

Incomplete kinetic decompositions avoid the uniqueness issue from the beginning. In the Ji decomposition [15], the gluon spin and OAM contributions are combined to form the gluon total angular momentum

$$\mathbf{J}_{\text{kin}}^G = \mathbf{S}^G + \mathbf{L}_{\text{kin}}^G = \int d^3r \mathbf{r} \times (\mathbf{E}^a \times \mathbf{B}^a) \quad (4)$$

which is local and therefore free from the Stueckelberg ambiguity. In the Belinfante decomposition, one further combines the quark spin and OAM contributions into the quark total angular momentum

$$\mathbf{J}_{\text{kin}}^q = \mathbf{S}^q + \mathbf{L}_{\text{kin}}^q = \int d^3r \bar{\psi} \mathbf{r} \times (\gamma^0 \frac{i}{2} \mathbf{D} + \boldsymbol{\gamma} \frac{i}{2} D^0) \psi \quad (5)$$

so that one can write $\mathbf{J}_{\text{kin}}^{q,G} = \mathbf{r} \times \mathbf{P}_{\text{kin}}^{q,G}$ with $P_{\text{kin}}^j = T_{\text{kin}}^{0j} = T_{\text{kin}}^{j0}$ where $T_{\text{kin}}^{\mu\nu}$ is the symmetric kinetic (or Belinfante-Rosenfeld) energy-momentum tensor. See Fig. 1 for a summary of the decompositions.

3 Spin sum rules and relations

Using the Belinfante-Rosenfeld version of the energy-momentum tensor, Ji obtained the remarkable result that the quark/gluon total kinetic angular momentum can be expressed in terms of twist-2 generalized parton distributions (GPDs) [15]

$$\langle J_{\text{kin}}^{q,G} \rangle = \frac{1}{2} \int dx x [H^{q,G}(x, 0, 0) + E^{q,G}(x, 0, 0)]. \quad (6)$$

This relation holds for the longitudinal component $J_L = \mathbf{J} \cdot \mathbf{P}/|\mathbf{P}|$ and does not depend on the magnitude of the proton momentum $|\mathbf{P}|$ [16, 21, 23]. By rotational symmetry, it is also valid for the transverse component, but only in the proton rest frame. Considering the transverse component of the Pauli-Lubanski vector does not prevent frame dependence of the separate quark and gluon contributions [22, 24, 13, 10].

Subtracting from Eq. (6) the longitudinal quark spin contribution, which is given by the isoscalar axial-vector form factor (FF) in the \overline{MS} scheme

$$\langle S^q \rangle = \frac{1}{2} G_A^q(0), \quad (7)$$

one obtains the following expression for the longitudinal quark kinetic OAM

$$\langle L_{\text{kin}}^q \rangle = \frac{1}{2} \int dx x [H^q(x, 0, 0) + E^q(x, 0, 0)] - \frac{1}{2} G_A^q(0). \quad (8)$$

The same quantitie can also be expressed in terms of a twist-3 GPD [45, 20, 12, 34]

$$\langle L_{\text{kin}}^q \rangle = - \int dx x G_2^q(x, 0, 0) \quad (9)$$

which appears in the longitudinal target spin asymmetry of deeply virtual Compton scattering [9].

The most intuitive expression for OAM is as a phase-space integral [35, 40]

$$\langle L^{q,G}(\mathcal{W}) \rangle = \int dx d^2k_{\perp} d^2b_{\perp} (\mathbf{b}_{\perp} \times \mathbf{k}_{\perp})_z \rho_{++}^{q,G}(x, \mathbf{k}_{\perp}, \mathbf{b}_{\perp}; \mathcal{W}), \quad (10)$$

Table 1 Results for the quark OAM from two light-front quark models for u , d and total ($u + d$) quark contributions.

Model q		LFCQM			LF χ QSM		
		u	d	Total	u	d	Total
$\langle L_{\text{kin}}^q \rangle$	Eq. (8)	0.071	0.055	0.126	-0.008	0.077	0.069
$\langle L_{\text{can}}^q \rangle$	Eq. (11)	0.131	-0.005	0.126	0.073	-0.004	0.069
$\langle L_{\text{can}}^q \rangle$	Eq. (12)	0.169	-0.042	0.126	0.093	-0.023	0.069

where the relativistic phase-space or Wigner distribution $\rho_{++}^{q,G}(x, \mathbf{k}_\perp, \mathbf{b}_\perp; \mathcal{W})$ can be interpreted as giving the (quasi-)probability for finding an unpolarized quark/gluon with momentum (xP^+, \mathbf{k}_\perp) at the transverse position \mathbf{b}_\perp inside a longitudinally polarized proton. In this semi-classical interpretation, the Euclidean subgroup of the light-front formalism plays a crucial role in providing a well-defined transverse center of the proton [46, 4, 5]. These phase-space distributions are related by Fourier transform to the so-called generalized transverse-momentum dependent distributions (GTMDs) [41, 39, 38], leading to the simple relation [35, 11, 19]

$$\langle L^{q,G}(\mathcal{W}) \rangle = - \int dx d^2 k_\perp \frac{\mathbf{k}_\perp^2}{M^2} F_{14}^{q,G}(x, 0, \mathbf{k}_\perp, \mathbf{0}_\perp; \mathcal{W}). \quad (11)$$

The type of OAM is determined by the shape of the Wilson line \mathcal{W} , namely $\langle L_{\text{kin}}^{q,G} \rangle = \langle L^{q,G}(\mathcal{W}_{\text{straight}}) \rangle$ and $\langle L_{\text{can}}^{q,G} \rangle = \langle L^{q,G}(\mathcal{W}_{\text{staple}}) \rangle$ [6, 30, 18]. Unfortunately, it is not known so far how to extract GTMDs from actual experiments, except perhaps at small x [41]. Interestingly, they are however in principle calculable on the lattice [17].

In the context of quark models, it has also been suggested that the quark canonical OAM could be related to a transverse-momentum dependent distribution (TMD)

$$\langle L_{\text{can}}^q \rangle = - \int dx d^2 k_\perp \frac{\mathbf{k}_\perp^2}{2M^2} h_{1T}^{\perp q}(x, \mathbf{k}_\perp^2), \quad (12)$$

but this relation is not valid in general in QCD [37] just like other relations among the TMDs [36]. In Table 1, the various expressions (8), (11) and (12) for the quark OAM are compared in two light-front quark models: the light-front constituent quark model (LFCQM) [1, 2, 42, 43, 44] and the light-front chiral quark-soliton model (LF χ QSM) [26, 27, 28]. While all the expressions agree for the total OAM, as they should, they differ in the flavor decomposition.

4 Spin-orbit correlation

What is referred to as the quark spin/OAM contribution to the proton spin corresponds more precisely to the *correlation* between the quark spin/OAM and the pro-

ton spin. There exists another interesting independent correlation characterizing the proton spin structure, although it does not appear in the proton spin decomposition, namely the correlation between the quark spin and the quark OAM. Like the OAM operators, one can define a kinetic and a canonical version of this spin-orbit correlation [35, 33]

$$\begin{aligned} \mathbf{C}_{\text{kin}}^q &= \int d^3x \psi^\dagger \gamma_5 (\mathbf{x} \times i\mathbf{D}) \psi, \\ \mathbf{C}_{\text{can}}^q &= \int d^3x \psi^\dagger \gamma_5 (\mathbf{x} \times i\mathbf{D}_{\text{pure}}) \psi. \end{aligned} \quad (13)$$

Like the average kinetic OAM contribution to the proton spin, the average quark longitudinal spin-orbit correlation can be expressed in terms of twist-2 and twist-3 GPDs [33]

$$\begin{aligned} \langle C_{\text{kin}}^q \rangle &= \frac{1}{2} \int dx x \tilde{H}^q(x, 0, 0) - \frac{1}{2} [F_1^q(0) - \frac{m_q}{2M_N} H_1^q(0)], \\ &= - \int dx x [\tilde{G}_2^q(x, 0, 0) + 2\tilde{G}_4^q(x, 0, 0)]. \end{aligned} \quad (14)$$

Remarkably, this shows that not only the first moment but also the second moment of the quark helicity distribution has physical interest.

The quark spin-orbit correlation can naturally also be expressed as a phase-space integral [35, 33]

$$\langle C^q(\mathcal{W}) \rangle = \int dx d^2k_\perp d^2b_\perp (\mathbf{b}_\perp \times \mathbf{k}_\perp)_z \rho_{++}^{[\gamma^+ \gamma_5]q}(x, \mathbf{k}_\perp, \mathbf{b}_\perp; \mathcal{W}), \quad (15)$$

where the relativistic phase-space distribution $\rho_{++}^{[\gamma^+ \gamma_5]q}(x, \mathbf{k}_\perp, \mathbf{b}_\perp; \mathcal{W})$ can be interpreted as giving the difference between the (quasi-)probabilistic distributions of quarks with polarization parallel and antiparallel to the longitudinal direction. In terms of the GTMDs, this relation reads [35, 19, 33]

$$\langle C^q(\mathcal{W}) \rangle = \int dx d^2k_\perp \frac{\mathbf{k}_\perp^2}{M^2} G_{11}^q(x, 0, \mathbf{k}_\perp, \mathbf{0}_\perp; \mathcal{W}). \quad (16)$$

Once again, the shape of the Wilson line \mathcal{W} determines the type of spin-orbit correlation, namely $\langle C_{\text{kin}}^q \rangle = \langle C^q(\mathcal{W}_{\text{straight}}) \rangle$ and $\langle C_{\text{can}}^q \rangle = \langle C^q(\mathcal{W}_{\text{staple}}) \rangle$ [33].

Because of the valence number constraints $F_1^u(0) = 2$ and $F_1^d(0) = 1$ and the small mass ratio $m_{u,d}/4M_N \sim 10^{-3}$, the essential non-perturbative input we need is the second moment of the quark helicity distribution

$$\int_{-1}^1 dx x \tilde{H}^q(x, 0, 0) = \int_0^1 dx x [\Delta q(x) - \Delta \bar{q}(x)]. \quad (17)$$

Contrary to the lowest moment $\int_{-1}^1 dx \tilde{H}_q(x, 0, 0) = \int_0^1 dx [\Delta q(x) + \Delta \bar{q}(x)]$, this second moment cannot simply be extracted from deep-inelastic scattering (DIS) polarized data. However, by combining inclusive and semi-inclusive DIS, separate quark and antiquark contributions can be extracted [25]. They can also be computed on

Table 2 Comparison between the lowest two axial moments for u and d quarks as predicted by various quark models, with the corresponding values obtained from the LSS fit to experimental data at $\mu^2 = 1 \text{ GeV}^2$ and lattice calculations at $\mu^2 = 4 \text{ GeV}^2$ and pion mass $m_\pi = 293 \text{ MeV}$.

Model	$\int dx \tilde{H}^u(x, 0, 0)$	$\int dx \tilde{H}^d(x, 0, 0)$	$\int dx x \tilde{H}^u(x, 0, 0)$	$\int dx x \tilde{H}^d(x, 0, 0)$
NQM	4/3	-1/3	4/9	-1/9
LFCQM	0.995	-0.249	0.345	-0.086
LF χ QSM	1.148	-0.287	0.392	-0.098
LSS [25]	0.82	-0.45	≈ 0.19	≈ -0.06
Lattice [3]	0.82(7)	-0.41(7)	≈ 0.20	≈ -0.05

the lattice [3]. In Table 2, the first two moments of the quark helicity distributions computed within the naive quark model (NQM), the LFCQM and the LF χ QSM are compared with the values obtained from inclusive and semi-inclusive DIS data [25] and from lattice calculations [3].

From these estimates, one obtains a negative kinetic quark spin-orbit correlation for both quark flavors, $\langle C_{\text{kin}}^u \rangle \approx -0.8$ and $\langle C_{\text{kin}}^d \rangle \approx -0.55$, meaning that in average the quark spin and kinetic OAM are expected to be antiparallel. On the contrary, the canonical version of the quark spin-orbit correlation appears to be positive in the models [35], showing the importance of the quark-gluon interaction.

5 Conclusion

There are essentially two types of proton spin decompositions: the kinetic one and the canonical one. It has recently been recognized that both are interesting and in principle measurable. The crucial missing piece in the proton spin decomposition is the contribution coming from the quark and gluon orbital angular momentum. Several relations and sum rules have been proposed in the literature, but few proved to be of practical significance. The current most promising approaches are based on the extraction of generalized parton distributions at twist 2 and 3 from experiments, and the direct calculation of orbital angular momentum on the lattice.

Another important aspect of the proton spin structure is the spin-orbit correlation which escaped attention until recently because it does not contribute to the proton spin decomposition. Like the orbital angular momentum, there are two types of spin-orbit correlations, and both are in principle measurable. This piece of information is of crucial importance if one aims at obtaining a complete description of the proton spin structure.

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